Damping

$\Delta > 0$ overdamped $\delta > 1$

$\Delta < 0$ underdamped $\delta < 1$

$\Delta = 0$ criticly damped $\delta = 0$

$\Delta = b^2 - 4ac$

$s^2 + 2\omega_0 \delta + \omega_0^2 = 0$

δ = type of damping

ω_0 = angular frequncy

α = damping coeficient

$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = F$

Chracteristic EQ:

$s^2 + 2\alpha s + \omega_0^2 = 0$ $\qquad \delta = \frac{\alpha}{\omega_0}$

$x(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t} + x(t\to\infty)$

$s = -\alpha \pm \sqrt{\alpha^2 - \omega^2} = \omega_0\left(-\delta \pm \sqrt{\delta^2 - 1}\right)$

$V_L(t)$ should always lead $I_L(t)$

$V_L(t)$ will always lead $I_L(t)$ by $90°$

$I(t) = 5\sin(420t - 20°)$A $\qquad V(t) = 10\cos(420t + 30°)$

$-20° + 90 = 70°$ leads V by $40°$

$V_C(t) = (V_C(0^-) - V_C(t\to\infty))e^{-t/\tau} + V_C(t\to\infty)$

$V_C(0^-) = V_C(0^+)$
$I_L(0^-) = I_L(0^+)$ $\Big\}$ Continuity Condition

Max Power Transfer

$= Z_{th}$
$=$ Max power transfer

$i(t) = |I|\cos(\omega t)$

$I_{rms} = \frac{|I|}{\sqrt{2}}$

$Z_{load} = Z_{th}$

$P_{avg} = \frac{1}{2}|I|^2 R \Rightarrow I_{rms}^2 R(\omega)$

$Z_C = \frac{1}{j\omega C}$

$Z_L = Lj\omega$

$Z_R = R$

Find impedence of Load?
Test source method
- turn off all sources
  - Ⓐ break Ⓥ wire
- input test Ⓥ over load
- $Z_{th} = \frac{V_T}{I_T} = V$impedence

IDEAL OP-AMP

$R_i \to \infty \quad R_o \to 0 \quad A \to \infty$

$\hookrightarrow \frac{V_o}{V_s} = -\frac{R_F}{R_s}$

$I_p = I_N = 0$

$\omega = 2\pi f$

$V(t) = V_A\cos(\omega_0 t + \phi)$

$V_A = \sqrt{a^2 + b^2}$ $\qquad a = V_a\cos(\phi)$, $b = V_a\sin(\phi)$

$\phi = \tan^{-1}\left(\frac{b}{a}\right)$

a and b $\Rightarrow$ $A\cos(\omega t) + B\sin(\omega t)$

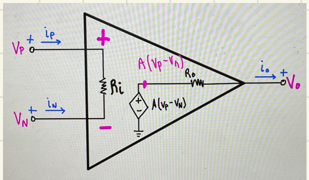
$\omega_c = \frac{1}{L}$ for inductors

$= 1/\sqrt{LC}$ for LC circuits

$\omega_c = \frac{1}{\tau}$

Inductor in series — high pass filter

Capacitor in series — low pass filter



$10^6 < R_I < 10^{12}\Omega$ LARGE INPUT RESISTANCE

$10 < R_o < 100\Omega$ SMALL OUTPUT RESISTANCE

$10^5 < A < 10^8$ LARGE VOLTAGE GAIN (OPEN LOOP GAIN)

QUALITY FACTOR Q_0

QUALITY FACTOR is the degree of selectivity of the circuit

HIGH Q $\longrightarrow$ NARROW BANDWIDTH & HIGH SELECTIVITY

$Q_0 \equiv \frac{\omega_0}{2\alpha}$ — ω_0: NATURAL FREQUENCY; α: NEPER FREQUENCY (AKA DAMPING FACTOR)

$Q_0 = \frac{1}{2\zeta}$ — ζ: DAMPING RATIO

OVERDAMPED	$Q_0 < \frac{1}{2}$	$\zeta > 1$
UNDERDAMPED	$Q_0 > \frac{1}{2}$	$\zeta < 1$
CRITICALLY DAMPED	$Q_0 = \frac{1}{2}$	$\zeta = 1$

TRANSFER FUNCTION

$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)}$ VOLTAGE GAIN TRANSFER FUNCTION

$|H(j\omega)| = \frac{\sqrt{(Re)^2 + (Im)^2}}{\sqrt{(Re)^2 + (Im)^2}}$ MAGNITUDE

$|H(j\omega)|_{dB} = 20\log_{10}(|H(j\omega)|)$ MAGNITUDE (decibels)

CONVERT FROM dB to VOLTS $\#V = 10^{\#dB/20}$

$\angle H(j\omega) = \frac{180°}{\pi}\tan^{-1}\left(\frac{Im}{Re}\right)$ PHASE (degrees)

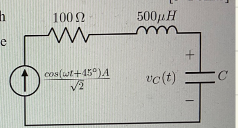
$P(j\omega) = H^2(j\omega)$

$P(j\omega_c) = \frac{H_0^2}{2}$

"-3dB" half power

Problem 2 - Frequency Response, Magnitude [5 Points]

a) [2.5pts] $V_C(j\omega)$ is resonant at 10krad/s with $|V_C(j\omega_0)| = 0.5V$ determine the numerical value of C in Farads.



$V_C(j\omega) = I_S(j\omega)Z_C \quad \to -\frac{j}{\omega C}\left(\frac{1}{2}\right)(1 + j) = \frac{1-j}{2(\omega C)}$

$|V_C(j\omega)| = \frac{\sqrt{2}}{2\omega C}$

$|V_C(j\omega)|_{\omega=\omega_0} = 0.5 = \frac{\sqrt{2}}{2(10krad/s)C}$

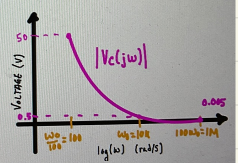
$C = \frac{1}{0.5 \times 10k} = 141\mu F$

b) [2.5pts] By hand, sketch $|V_C(j\omega)|_V$

$\left|V_C\left(\frac{j\omega_0}{100}\right)\right|_{\omega<<\omega_0} = \frac{1}{\frac{10k}{100}(200\mu F)} \to 50V$

$|V_C(j\omega_0)|_{\omega=\omega_0} = \frac{1}{10k(200\mu F)} \to 0.5V$

$|V_C(j100\omega_0)|_{\omega>>\omega_0} = \frac{1}{10k(200\mu F)} \to 0.005V$



Problem 3 - Saturation

a) Derive an expression for the Dynamic Linear Range of the output voltage in terms of $\pm V_{CC}$, V_{s1}, V_{s2}, and the resistors. **Recall:** $-V_{CC} \leq V_o \leq +V_{CC}$ $\quad |\pm V_{CC}| = 15V$

LINEAR RANGE of OPERATION for OPAMP

$-15V \leq \left(\frac{10}{2}\right)(V_{S1} + 1) \leq +15V$

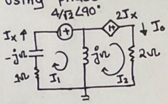
$-15V \leq 5V_{S1} + 5 \leq +15V$

$-20V \leq 5V_{S1} \leq 10V$

$-4V \leq V_{S1} \leq 2V$

Mesh with dependent source

Using phasors



KVL:

I_1 : $I_1 - jI_1 - j\frac{4}{\sqrt{2}} + j2(I_1 - I_2) = 0$

I_2 : $j2(I_2 - I_1) - 2J_x + 2I_2 = 0$

Aux: $I_1 = J_x \quad I_2 = I_o$

After solving the 2 eqn:

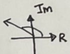
$(1-j)\,I_o = j\,4/\sqrt{2}$

$I_o = j\frac{4}{\sqrt{2}}\left(\frac{1}{1-j}\right) \times \left(\frac{1+j}{1+j}\right)$

$= j\,\frac{4}{\sqrt{2}}\left(\frac{1+j}{2}\right)$

$= \frac{2}{\sqrt{2}}(j-1)$

$|I_o| = \frac{2}{\sqrt{2}} \times \sqrt{2} = 2.$

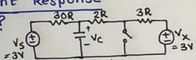


$\angle I_o = 135^\circ \quad [90+45]$

$I_o(t) = 2\cos(\omega t + 135).$

Transient Response

$V_c(t) = ?$

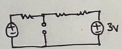


$V_c(t) = V_c(0^+) - V_c(\infty)\, e^{-t/R_{th}C} + V_c(\infty)$

$V_c(0^-) = V_c(0^+)$

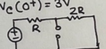
At DC

$V_c(0^-)$:



$V_c(0^-) = V_c(0^+) = 3V$

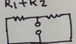
$V_c(t\to\infty)$:



$V_c(\infty) = 3V\left(\frac{2R}{R+2R}\right) = 2V.$

$R_{11} = \frac{R_1 R_2}{R_1 + R_2}$

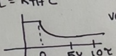
R_{TH} :



$R_{TH} = \frac{2R^2}{R+2R}$

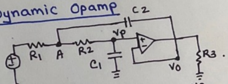
$= \frac{2}{3}R = 20\Omega.$

$\tau = R_{TH}C$



$v(t) = (e^{-t/0.2} + 2)\,V$

Dynamic Opamp



$A: \left(\frac{1}{R_1} + \frac{1}{R_2} + j\omega C_2\right)A - \frac{V_S}{R_1} - \frac{V_P}{R_2} - V_o\, j\omega C = 0$

$V_P: \left(\frac{1}{R} + j\omega C_1\right)V_P - \frac{A}{R_2} = 0$

$V_N = V_P = V_o$

↳ negative feedback

$\frac{V_o}{V_S} = H(j\omega)$

NOTE

RLC standard form:

$s^2 + 2\omega_0 \zeta s + \omega_0^2 = 0$

$\omega_0 = \sqrt{1/LC} = \frac{1}{\sqrt{LC}}$

* voltage leads – inductor
current leads – capacitor.

Resonance, ω_0

$X_L = X_C$

$j\omega_0 L = \frac{1}{j\omega_0 C}$

$\omega_0^2 = \frac{1}{LC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$

$dB_V = 20\log_{10}(|H(j\omega)|)$

@ DC

R:

C: ⇒ open

L: ⇒ short

@ AC

C: $V_c(0^-) = 0V$

$Z_C = \frac{1}{j\omega C}$

$\omega \to \infty$

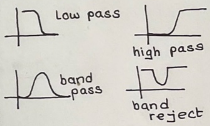
$Z_C \to 0$

L: $Z_L = j\omega L$

$\omega \to \infty$

$Z_L \to \infty$

open



@ t≤0 : C ⇒ open, L ⇒ short

@ t = ∞ : C ⇒ open, L ⇒ short

1st order

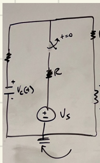
For capacitor:

$V_c(t) = [V_c(0^-) - V_c(t\to\infty)]\,e^{-t/\tau} + V_c(t\to\infty)\,V \qquad \tau = R_{TH}C$

For inductor:

$i_L(t) = [i_L(0^+) - i_L(t\to\infty)]\,e^{-t/\tau} + i_L(t\to\infty)$

$\tau = \frac{L}{R_{TH}}$



$I_c = C\frac{dV_c}{dt}$

$V_L = L\frac{dI_L}{dt}$

$-I_L = I_c$

$I_1 = I_c = C\frac{dV_c}{dt}$

$-V_c(t) + I_1R + I_1R + V_c(t) = 0$

$-V_c(t) + 2RC\frac{dV_c}{dt} + L\frac{dI_L}{dt} = 0$

$-V_c(t) + 2RC\frac{dv}{dt} + LC\frac{d^2V_c}{dt^2} = 0$

$\frac{d^2V_c}{dt} + \frac{2R}{2L} + \frac{V_c(t)}{2LC} = 0$

$r^2 + \frac{R}{L}r + \frac{1}{2LC} = 0$

$s^2 + \delta\omega_0 + \omega_0^2$

$\delta = \frac{R}{L}\sqrt{2LC} \qquad \omega_0 = \frac{1}{\sqrt{2LC}}$

Initial Conditions

$V_c(0^-) = V_c(0^+)$

$I_L(0^-) = I_L(0^+)$

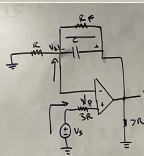
$V_c(0^-) = 5V = V_c(0^+)$

$I_L(0^-) = \frac{10V}{2R} = 500\mu A = I_L(0^+)$

10kΩ

Final conditions

$V_c(t\to\infty) = 0$



$V_c(t) = ?$

C initialy discharged

$V_P = V_N = V_S \qquad V_c = V_o - V_S$

$V_N: -\frac{V_N}{R} + \frac{V_o - V_N}{R_F} + I_c = 0$

$-\frac{V_S}{R} + \frac{V_c}{R_F} + C\frac{dV_c}{dt} = 0$

$I_c = \frac{dV_c}{dt}C$

$V_c = B$

$V_c' = 0$

Particular

$\frac{dV_c}{dt} + \frac{V_c}{CR_F} = \frac{V_S}{R_C}$

$0 + \frac{B}{CR_F} = \frac{V_S}{R_C}$

$B = \frac{V_S(CR_F)}{R_C}$

$0 = Ke^{-0/R_FC} + \frac{V_S(CR_F)}{R_C}$

$V_c(t) = \frac{V_S(CR_F)}{R_C}\left(-e^{-t/R_FC} + 1\right)$

$\frac{dV_c}{dt} + \frac{V_c}{CR_F} = \frac{V_S}{R_C}$

homogeneous

trial Ke^{st}

ske^{st}

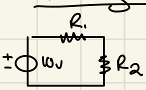
$ske^{st} + \frac{ke^{st}}{CR_F} = 0$

$ke^{st}\left(s + \frac{1}{CR_F}\right) = 0$

$s = -\frac{1}{CR_F}$

Ke^{-t/R_FC}

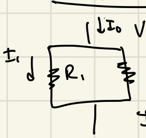
Voltage Devider



V_o drop acuross R_1

$V_{R_1\,drop} : 10v \cdot \frac{R_1}{R_1 + R_2}$

Current Devider



$I_1 = I_o\left(\frac{R_2}{R_1 + R_2}\right)$